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Forecasting in R

Evaluating modeling accuracy

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Outline

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Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

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Assumptions

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2. $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

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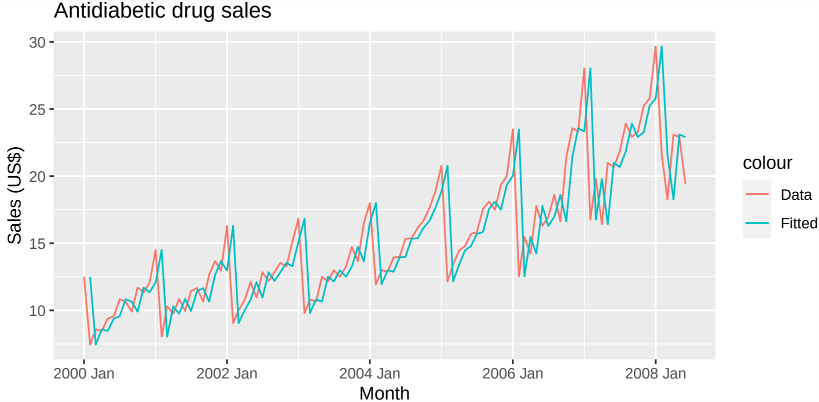
Useful properties (for prediction intervals)

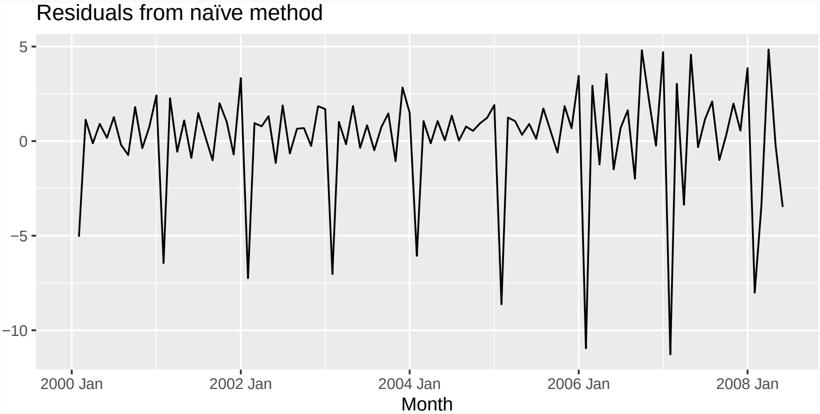
3. $\{e_t\}$ have constant variance.
4. $\{e_t\}$ are normally distributed.

Example: Antidiabetic drug sales



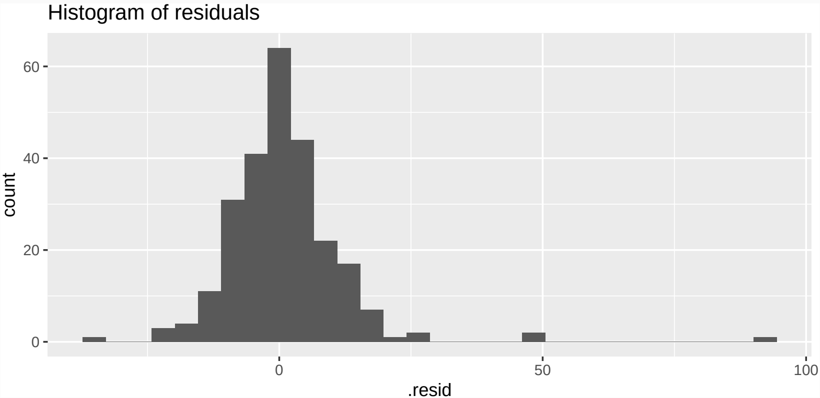
Example: Antidiabetic drug sales

```r
augment(fit) %>%
  autoplot(.resid) + xlab("Month") + ylab("") +
    ggtitle("Residuals from naïve method")
```



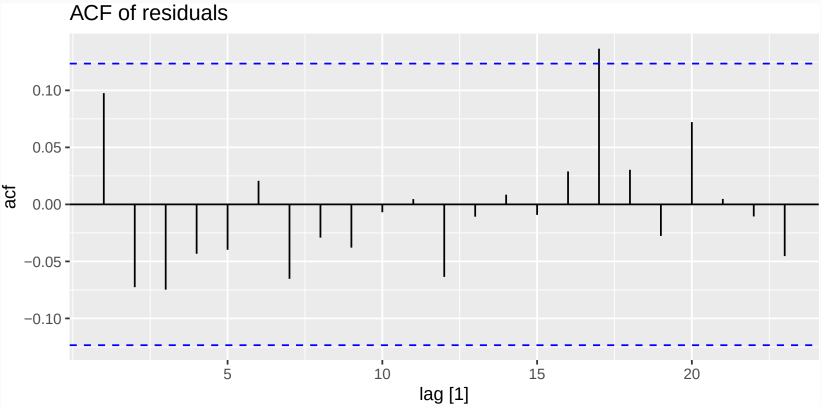
Example: Antidiabetic drug sales

```r
augment(fit) %>%
  ggplot(aes(x = .resid)) +
    geom_histogram(bins = 30) +
    ggtitle("Histogram of residuals")
```



Example: Antidiabetic drug sales

```r
augment(fit) %>% ACF(.resid) %>%
  autoplot() + ggtitle("ACF of residuals")
```



ACF of residuals

- We assume that the residuals are white noise (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.
- We *expect* these to look like white noise.

Portmanteau tests

Consider a *whole set* of r_k values, and develop a test to see whether the set is significantly different from a zero set.

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Box-Pierce test

$$Q = T\sum_{k=1}^{h} r_k^2$$

where h is max lag being considered and T is number of observations.

- If each r_k close to zero, Q will be **small**.
- If some r_k values large (positive or negative), Q will be **large**.

Portmanteau tests

Consider a *whole set* of r_k values, and develop a test to see whether the set is significantly different from a zero set.

Ljung-Box test

$$Q^* = T(T+2)\sum_{k=1}^{h}(T-k)^{-1}r_k^2$$

where h is max lag being considered and T is number of observations.

- Preferences: $h = 10$ for non-seasonal data, $h = 2m$ for seasonal data.
- Better performance, especially in small samples.

Portmanteau tests

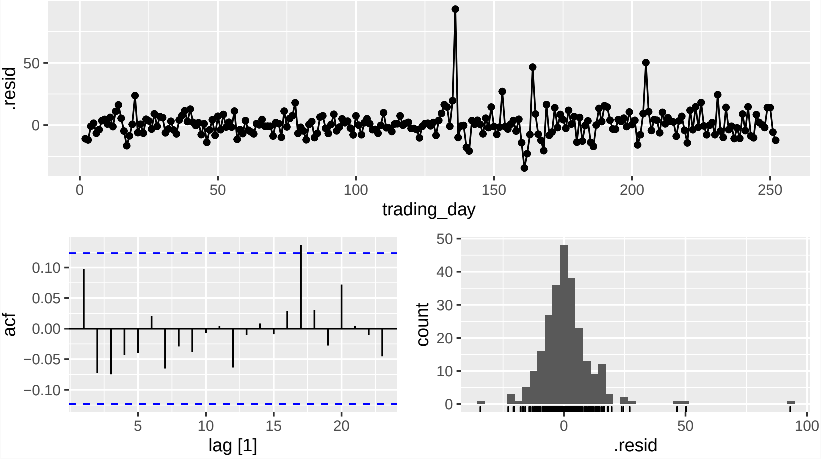
- If data are WN, Q^* has χ^2 distribution with $(h - K)$ degrees of freedom where K = no. parameters in model.
- When applied to raw data, set $K = 0$.

```
augment(fit) %>% features(.resid, ljung_box, lag=10,dof=0)
```

```
## # A tibble: 1 x 4
##   Symbol .model        lb_stat lb_pvalue
##   <chr>  <chr>           <dbl>     <dbl>
## 1 GOOG   NAIVE(Close)     7.91     0.637
```

`gg_tsresiduals` function

```r
fit %>%
  gg_tsresiduals()
```



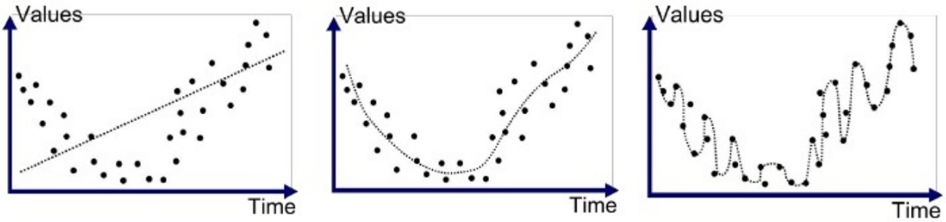
Outline

Evaluating point forecast accuracy

Evaluate forecast accuracy

- Residual diagnostic is not a reliable indication of forecast accuracy
- A model which fits the training data well will not necessarily forecast well
- A perfect fit can always be obtained by using a model with enough parameters
- Over-fitting a model to data is just as bad as failing to identify a systematic pattern in the data

Fitting



Underfitted

Good Fit/Robust

Overfitted

Evaluate forecast accuracy

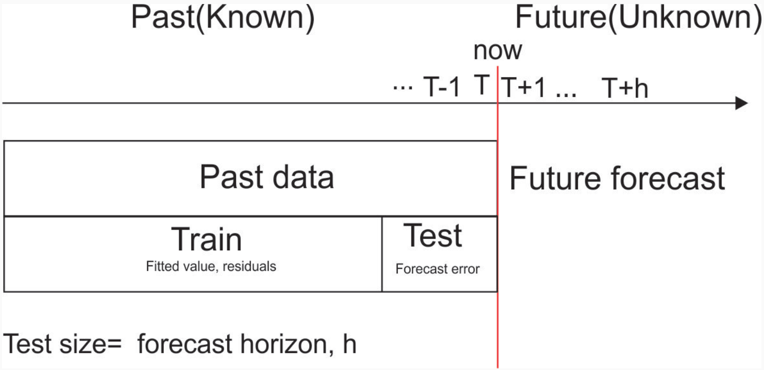
The accuracy of forecasts can only be determined by considering how well a model performs on new data that were not used when fitting the model

Forecast accuracy evaluation using test sets

- We mimic the real life situation
- We pretend we don’t know some part of data(new data)
- It must not be used for *any* aspect of model training
- Forecast accuracy is based only on the test set

Outline

Training and test series



Split the data

Use functions in `dplyr` and `lubridate` such as `filter`, `filter_index`, `slice`, `year`

```
# Filter the year of interest
antidiabetic_drug_sale %>%
  filter_index("2006"~.)
```

```
## # A tsibble: 30 x 2 [1M]
##       Month  Cost
##       <mth> <dbl>
##  1 2006 Jan  23.5
##  2 2006 Feb  12.5
##  3 2006 Mar  15.5
##  4 2006 Apr  14.2
```

Forecast errors

Forecast "error": the difference between an observed value and its forecast

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T},$$

where the training data is given by $\{y_1, \dots, y_T\}$

- Unlike residuals, forecast errors on the test set involve multi-step forecasts.
- These are *true* forecast errors as the test data is not used in computing $\hat{y}_{T+h|T}$.

Measures of forecast accuracy

$$
\begin{aligned}
y_{T+h} &= (T+h)\text{th observation, } h = 1, \dots, H \\
\hat{y}_{T+h|T} &= \text{its forecast based on data up to time } T. \\
e_{T+h} &= y_{T+h} - \hat{y}_{T+h|T}
\end{aligned}
$$

$$
\begin{aligned}
\text{MAE} &= \text{mean}(|e_{T+h}|) \\
\text{MSE} &= \text{mean}(e_{T+h}^2) \qquad\qquad \text{RMSE} = \sqrt{\text{mean}(e_{T+h}^2)} \\
\text{MAPE} &= 100\text{mean}(|e_{T+h}|/|y_{T+h}|)
\end{aligned}
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\end{aligned}
$$

- MAE, MSE, RMSE are all scale dependent
- MAPE is scale independent but is only sensible if $y_t \gg 0$ for all t, and y has a natural zero.

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|e_{T+h}|/Q)$$

where Q is a stable measure of the scale of the time series $\{y_t\}$.

For non-seasonal time series,

$$Q = (T-1)^{-1}\sum_{t=2}^{T}|y_t - y_{t-1}|$$

works well. Then MASE is equivalent to MAE relative to a naïve method.

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|e_{T+h}|/Q)$$

where Q is a stable measure of the scale of the time series $\{y_t\}$.

For seasonal time series,

$$Q = (T-m)^{-1} \sum_{t=m+1}^{T} |y_t - y_{t-m}|$$

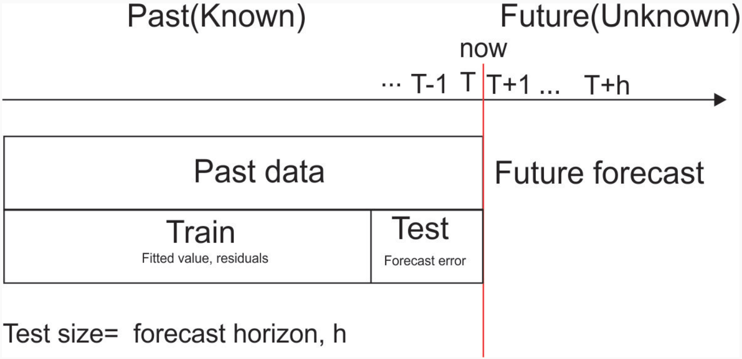
works well. Then MASE is equivalent to MAE relative to a seasonal naïve method.

Poll: true or false?

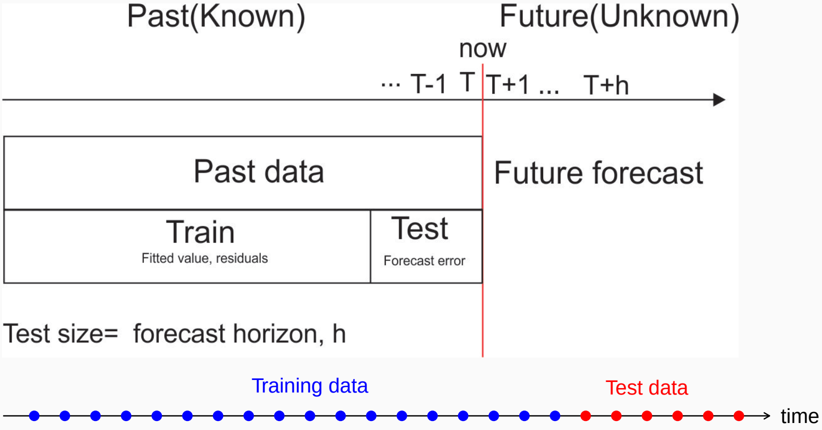
1. Good point forecast models should have normally distributed residuals.
2. A model with small residuals will give good forecasts.
3. The best measure of forecast accuracy is MAPE.
4. Always choose the model with the best forecast accuracy as measured on the test set.

Outline

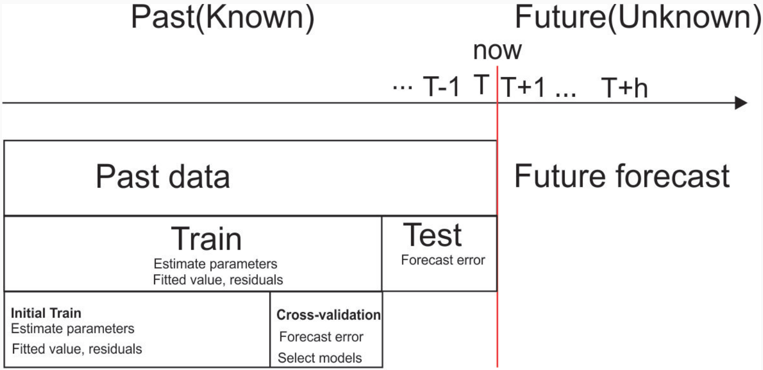
Issue with traditional train/test split



Issue with traditional train/test split



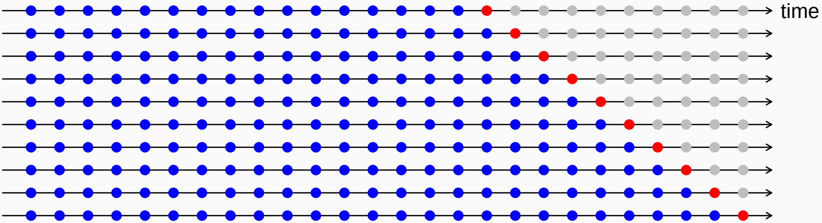
Time series cross-validation



Test size= forecast horizon, h

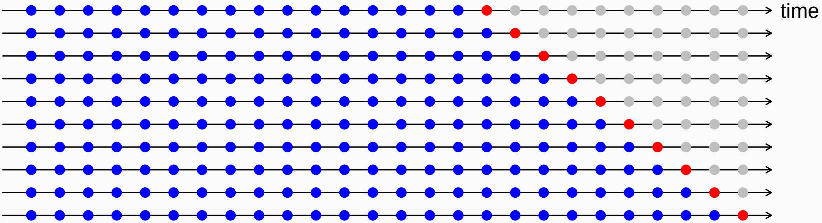
Cross-validation size=nb of experiment+h-1

Time series cross-validation



Time series cross-validation

Time series cross-validation



- Forecast accuracy averaged over test sets.
- Also known as “evaluation on a rolling forecasting origin”

Creating the rolling training sets

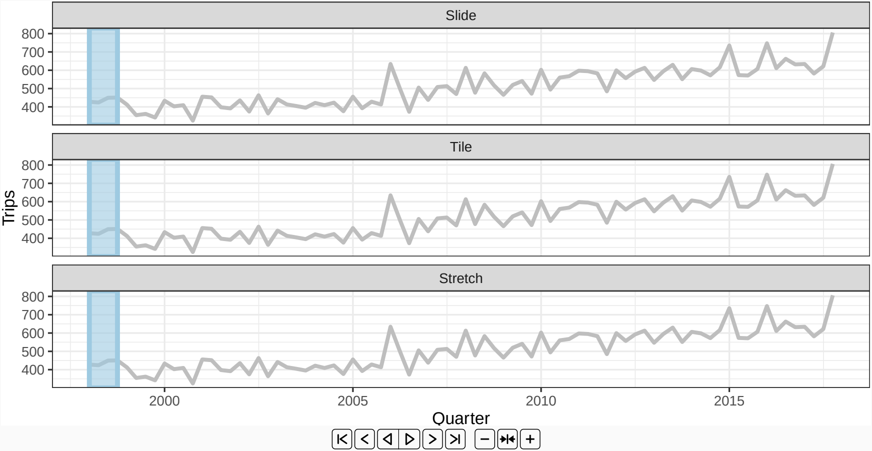
There are three main rolling types which can be used.

- Stretch: extends a growing length window with new data.
- Slide: shifts a fixed length window through the data.
- Tile: moves a fixed length window without overlap.

Three functions to roll a tsibble: `stretch_tsibble()`, `slide_tsibble()`, and `tile_tsibble()`.

For time series cross-validation, stretching windows are most commonly used.

Creating the rolling training sets



Time series cross-validation

Stretch with a minimum length of 24, growing by 1 each step.

```
forecast_horizon <- 12
test <- antidiabetic_drug_sale %>%
  slice((n()-forecast_horizon+1):n())
train <- antidiabetic_drug_sale %>%
  slice(1:(n()-forecast_horizon))
drug_sale_tcsv <-  train %>% slice(1:(n()-forecast_hori
  stretch_tsibble(.init = 24, .step = 1)
```

```
## # A tsibble: 2,805 x 3 [1M]
## # Key:       .id [55]
##      Month  Cost   .id
##      <mth> <dbl> <int>
## 1 2000 Jan 12.5      1
## 2 2000 Feb  7.46     1
```

Time series cross-validation

Estimate RW w/ drift models for each window.

```
drug_fit_tr <- drug_sale_tcsv %>%
  model(snaive=SNAIVE(Cost))
```

```
## # A mable: 55 x 2
## # Key:     .id [55]
##      .id   snaive
##    <int>  <model>
## 1      1 <SNAIVE>
## 2      2 <SNAIVE>
## 3      3 <SNAIVE>
## 4      4 <SNAIVE>
## # ... with 51 more rows
```

Time series cross-validation

Produce 8 step ahead forecasts from all models.

```
drug_fc_tr <- drug_fit_tr %>%
  forecast(h=forecast_horizon) %>%
  group_by(.id) %>%
  mutate(h=row_number()) %>%
  ungroup()
```

```
## # A fable: 660 x 6 [1M]
## # Key:     .id, .model [55]
##      .id .model    Month         Cost .mean     h
##    <int> <chr>     <mth>       <dist> <dbl> <int>
## 1      1 snaive 2002 Jan  N(14, 1.7) 14.5      1
## 2      1 snaive 2002 Feb   N(8, 1.7)  8.05     2
## 3      1 snaive 2002 Mar  N(10, 1.7) 10.3      3
## 4      1 snaive 2002 Apr N(9.8, 1.7)  9.75     4
```

Time series cross-validation

```r
# Cross-validated
drug_fc_tr %>% accuracy(antidiabetic_drug_sale)
```

Outline

Winkler score

Winkler proposed a scoring method to enable comparisons between prediction intervals:

- it takes account of both coverage and width of the intervals.

Winkler score

$$W(l_t, u_t, y_t) = \begin{cases} u_t - l_t & \text{if } l_t < y_t < u_t \\ (u_t - l_t) + \frac{2}{\alpha}(l_t - y_t) & \text{if } y_t < l_t \\ (u_t - l_t) + \frac{2}{\alpha}(y_t - u_t) & \text{if } y_t > u_t \end{cases}$$

Prediction interval accuracy

```
 # Compute interval accuracy
drug_fc_tr %>%
 accuracy(antidiabetic_drug_sale,
  measures = interval_accuracy_measures) %>%
  mutate(Method = paste(.model, "method")) %>%
 select(Method, winkler) %>%
 gt::gt() %>%
 gt::as_latex()
```

Method	winkler
snaive method	9.731097

Outline

Lab session 6

Compute seasonal naïve forecasts for daily A&E n_attendance:

1. use `slice()` function to subset data into train and test
   - keep the last 42 days for test set
2. Speicify model and train data on train set
3. visualise forecasts
4. Test if the residuals are white noise.
   - use `gg_tsdisplay` function and `Lj` test
   - What do you conclude?

Lab session 6

5. Create folds/windows for time series cross validation
   - **Hint:** use `stretch_tsibble(.init = 4*365, .step = 1)`
6. Train model on each fold/window
7. Forecast for 42 days
8. Compute RMSE and MAE to evaluate point forecast
9. Evaluate the prediction intervals using Winkler score.

Recap

1. First, import your data and prepare them using `tsibble` function.
2. Visualise and see wether your series contains key features
3. Determine how much of your data you want to allocate to training, and how much to testing; the sets should not overlap.
4. Subset the data to create a training set, which you will use as an argument in your forecasting function(s). Optionally, you can also create a test set to use later.
5. Compute forecasts of the training set using whichever forecasting function(s) you choose, and set h equal to the number of values you want to forecast.

Recap

6. Use residual diagnostic based on residuals in the training set to see wether all informations is captured by models.
7. Create different windows to evaluate forecast accuracy using time series cross validation
8. Train model to each window
9. To view the results of accuracy, use the accuracy() function with the `fable` as the first argument and original data as the second.
10. Pick a measure in the output to evaluate the forecast(s); a smaller error indicates higher accuracy.
11. Forecast using all data for test set and visualise forecasts against actual values
12. Finally, produce forecast using the selected approach for future.